Quantum Composer Challenge: The Triple-well potential

Background

The Triple-Well potential

The triple-well potential has been studied for many years as an atomtronics analogue to a transistor [1, 2, 3, 6]. Atomtronics is subfield of atomic physics wherein atoms take the place of electrons. Thus, the chemical potential of trapped atoms is analogous to electrical potential, atom flux is analogous to current, and potential barriers can be thought of as resistive elements.

Atomtronic transistors are interesting in that they can produce gain. That is, a small atom potential in a central well (the *gate* well) can be used to control the population transfer of a larger atom potential from a *source* well to a *drain* well (or vice-versa). This is analogous to how a field-effect transistor works in solid-state electronics.

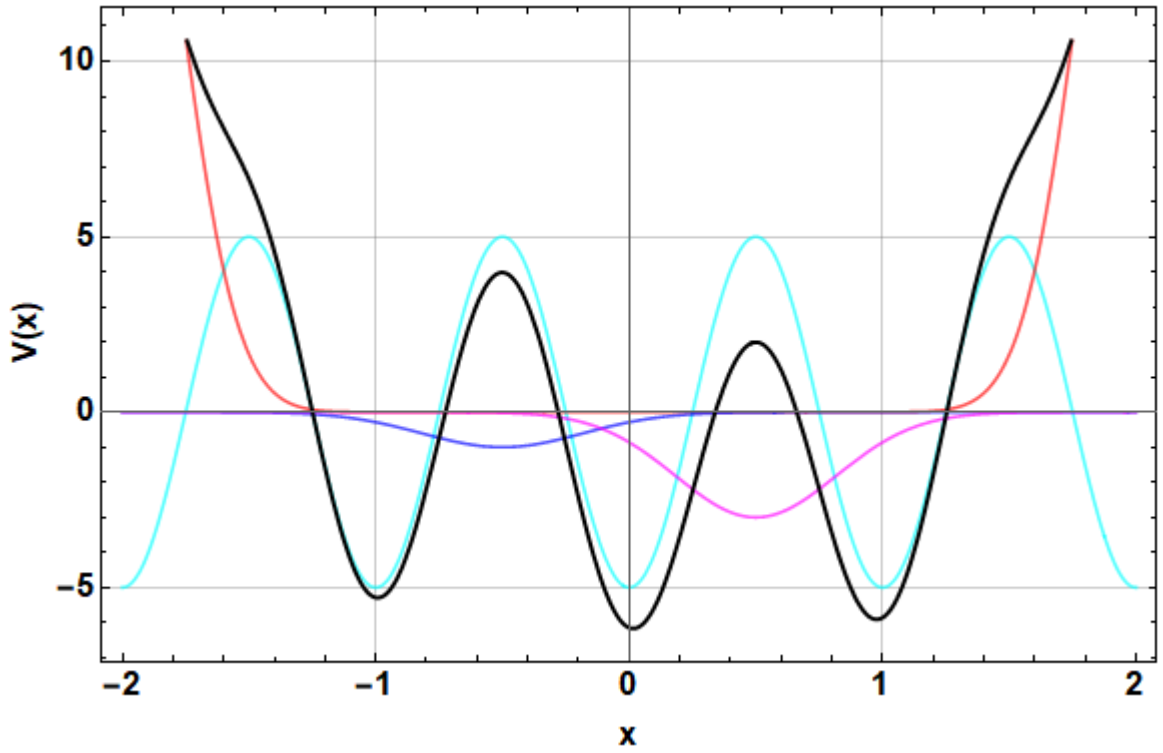

Figure 1: The triple-well potential you will be exploring in this challenge. The overall potential (black) is the sum of four different potentials: an optical lattice potential (cyan), a repulsive potential (red) created by summing two Gaussian barriers, and two attractive Gaussian potentials (blue and magenta) representing the laser beams that control the heights of the barriers between the three wells. The amplitudes of the two attractive Gaussian potentials are your controls.

The system that you will be exploring today (Fig. 1) is a simplification of the models presented in [2, 3], but it is interesting in its own right. You will be given two version of a triple-well potential created from an optical lattice potential modified by two repulsive Gaussian potentials to limit the dynamics to three wells. Additionally, the heights of the barriers are modulated by two Gaussian laser beams that can lower (but not raise!) the heights between the barriers. Thus, the full potential (neglecting the two repulsive

potentials) takes the form

$$V(x,t) = -a\cos(2\pi x)/2 + A_1(t)\exp\big(-b(x-x_0)^2\big) + A_2(t)\exp\big(-b(x+x_0)^2\big) \quad (1)$$

where we have maintained consistency with the Composer variables as much as possible. The variable a denotes the lattice depth, and the lattice is set up so that the sites exist at $x = 0$ and ± 1. The lasers that modulate the barriers are designed so that $b = 5$ and in order to center them between the lattice sites, $x_0 = 0.5$. The amplitudes A_1 and A_2 of these lasers (called u_0 and u_1 in the control problems) are time-dependent and control the height of the barriers between lattice sites. **These amplitudes cannot go negative!** The idea is that, by modulating the barriers between the wells, the transfer of the wavefunction from the first well to the third well can be done more quickly than without this modulation.

Your challenge is to understand how the system evolves as you change the barrier heights. From this, you should be able to come up with a sequence of barrier height manipulations that transport the wavefunction as quickly as possible from the left- to the right-most well.

Quantum control of the system

You can attack this problem in three ways. First, you will be able to define explicit functional forms for the barrier modulation functions and examine how these affect the system dynamics. Additionally, you can control the barrier modulation by use of two interactive graphs that can be manipulated by clicking and dragging different points. Finally, you will be able to use the interactive graph to build an initial guess for a gradient-based optimization algorithm called GRAPE that is a workhorse of quantum control [4].

In many ways, this challenge is open-ended in that we do not know what the "right" answers are. We will provide you with a sketch for how you should go through the flowscenes and some guiding questions, but the results and their interpretation will largely be up to you.

Nonlinearity and Bose-Einstein Condensation

Differences arise when one considers the control of a Bose-Einstein Condensate (or BEC) compared to a single-particle wavefunction. A BEC is a very cold cloud of bosonic atoms that can be used to study a wide variety of physical phenomena. The literature on Bose-Einstein condensation is voluminous, but one can start with Refs. [5, 8] and references therein.

In particular, BECs are interesting for atomtronics because of their interesting properties, including (but not limited to!) a non-linear atom-atom interaction that arises when we consider the system in a mean-field theory (that is, a theory that averages out complex dynamics to give a single effective interaction parameter). This gives rise to interesting and non-trivial dynamics when considering the transistor-like geometry we consider here.

This is best illustrated by looking at the form of the two Hamiltonians that describe a linear system (governed by the Schrödinger equation) and the nonlinear system governing mean-field BEC dynamics. For a given potential $V(x,t)$ as in Eq. (1), the single-particle Hamiltonian $H(x,t)$ takes the familiar form

$$H(x,t) = \frac{p^2}{2m} + V(x,t). \quad (2)$$

However, if we add the nonlinearities due to the interactions of ultracold bosonic atoms with one another and make the mean-field approximation (that is reasonably good in most situations), we obtain the so-called Gross-Pitaevskii equation (GPE) [7]

$$H(x,t) = \frac{p^2}{2m} + V(x,t) + \beta|\psi(x,t)|^2, \quad (3)$$

where the parameter β controls the atom-atom interaction. This parameter is dependent on the atom density, the atom-atom scattering length, and the atomic mass. This nonlinearity is interesting in that it modifies the effective potential seen by the atoms, and as a result, controls that work for a single particle may fail when one considers a BEC.

The flowfiles

Here, you are provided with two sets of three flowfiles. Both sets describe the triple-well potential, and the only difference is that one set solves the linear, single-particle equation Eq. (2) and the other solves the nonlinear equation Eq. 3. As described in the previous section, for each of these, you will be able to control the potential modulation via

- A graph with points that you can click-and-drag.
- A function that you write directly into Composer.
- Optimal control with the GRAPE algorithm.

In this challenge, please only change certain aspects of the potential. That is, you will not change most of the values that go into the *Potential* nodes. You may explore what changes as you modulate the lattice depth a, and you may change the amplitudes of the barriers ($u_0(t)$ and $u_1(t)$) If you change the potential parameters (other than the aforementioned exception), we cannot guarantee that you are simulating the system properly, as we have carefully taken into account what is physical based on how Composer handles boundary conditions in static and dynamic problems. Additionally, when exploring the BEC levels, you may change the values of the nonlinearity parameter β. Finally, you can (and should!) change the final time dt. For simplicity, we list the parameters that may be changed and how they are typically represented in Composer:

- The amplitudes $u_0(t)$ and $u_1(t)$
- The lattice depth a
- The final control time T and the temporal resolution dt
- The BEC nonlinearity parameter β (where applicable!)
- The control graph or control function

All of these parameters should be modified outside of the *Potential* and *Control* nodes. You should not be modifying the text in those nodes directly.

Syntax in the *Scalar Expression* nodes that define the amplitude modulation functions is like that used in most programming languages. If you cannot get your favorite common function to work, check the Reference page on the Composer website (particularly, the page on Composer basics).

Below, we give you some guiding questions to explore as you work through the challenge.

The challenge

Below we provide a list of questions and rough order in which to explore them. Note however that you cannot answer all of these questions in your report, so you should pick an interesting thread or two and explore them in detail. We much prefer an in-depth analysis of a few compelling bits of the problem than a shallow analysis of many aspects of the problem. We have tried to make the system extremely flexible, and some of the threads you explore might not lead to anything extremely interesting. That's okay! Take a step back and re-formulate your strategies, then pick another thread to tug on.

1. Start with the single-particle flowscenes. Play with the system using the draggable graph and the function input scenes. Don't use the GRAPE optimization flowscenes yet. Consider the following questions:
    - What is transfer time from the left to right well that you can achieve if you do not modulate the barrier at all? (Define the transfer time as the first time where the transfer fidelity is $F > F_T$ for some threshold fidelity F_T.) This defines a base transfer time $T_0(F_T)$ that is a function of F_T.
    - Can you improve this transfer time by modulating the barriers? That is, can you improve the transfer time to $T_{\text{new}} < T_0$? If so, how does this improvement scale as F_T changes?

- How does your improved transfer time depend on F_T? Map out a T_{new} vs. F_T curve. At what point does $T_{\text{new}}(F_T) = T_0(F_T)$
- Which control method is easier for you to work with? Why?
- What strategies work for each case? Which do not?

2. Now play with the same flowscenes constructed for the BEC problem. Consider:
   - Does $T_0(F_T)$ change? What is $T_{0,\text{BEC}}$ as a function of F_T and β?
   - Do the same controls work for the BEC as for the single-particle?
   - How does your answer to the above change as you change β?
   - Is this system easier or harder to control when working with a BEC?
   - Map out the $T_{\text{new,BEC}}$ vs. F_T curve for various values of β.
   - What happens if $\beta < 0$? What does this correspond to physically? (You may need to do some research to figure out the physical interpretation. Start by looking at the citations provided above.)
3. Once you have given the non-GRAPE flowscenes a try, move on to the GRAPE-based flowscenes.
   - Explore the same questions as above, but using the GRAPE optimizer. In particular, what strategies does the GRAPE optimizer seem to use? That is, how do the barriers behave for the optimal protocols? How does the T_{new} vs. F_T curve differ when working with the GRAPE algorithm? How does the curve vary as you change β in the BEC case?
   - What happens if you start with some of the wavefunction in the center well? Does this change as you move between the single-particle and BEC case?
   - How are controls different for the single-particle and BEC case, given a similar final time?
   - How do the GRAPE results depend on the initial guess?

Reporting your results

As stated in the overview, please write up your most interesting results in a cohesive 2-4 page report. Describe your work and include pictures! A LaTeXtemplate will be provided for you.

Remember: we don't know the exact answers to many of these questions, so part of the game is to teach us something! In many ways, this is truly original research.

Finally, please think about the following: Are there any other problems you would like to explore in Composer? We are always open to new and exciting ideas!

References

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