Shaking and Squeezing the Quantum Harmonic Oscillator

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Background

The simple harmonic oscillator potential is ubiquitous in quantum mechanics, and it is a reasonable approximation for many real-world potentials studied in laboratories around the world. The potential is characterized by a particle mass m and a harmonic frequency ω

$$V_{\text{SHO}}(x) = \frac{1}{2}m\omega^2 x^2 \tag{1}$$

admits eigenstates with energies given by

$$E_n = \hbar\omega(n + 1/2). \tag{2}$$

Remember that in Composer, the quantities $\hbar$ and m are both set to unity!

For example, we can put a particle (like an atom) into the ground state of a harmonic potential. However, in some cases, it is useful to be able to excite the atom to higher excited states. We can do this by modulating the potential. This exercise will do just that: we will modulate the potential in two different ways, and the behavior of the quantum wavefunction will be analyzed in both cases.

Note that in atomic physics, we call these techniques variations on **modulation spectroscopy**, and that's where the flowfiles get their names.

The flowfile

In these cases, we have built the flowscenes for you, but from what you have learned in the Spectrum and Correlation Amplitude exercises, you should be able to understand the code. They are called

- `modulation_spectroscopy_shake_2019.flow` and

- `modulation_spectroscopy_squeeze_2019.flow`.

For each flowscene, there are *Scalar* nodes on the left that you can change. These numbers define the time evolution and perturbation parameters that will be described in more detail in the problem section. On the right, the calculations are done.

In the top panel, we find code that calculates the ground and first five excited eigenstates of the harmonic oscillator potential. In the bottom panels, we apply the modulation via a time-dependent Hamiltonian via a *Time Evolution* node placed within a *For Loop* scope.

1 Problem 1: Shaking the Harmonic Oscillator

Open the file `modulation_spectroscopy_shake_2019.flow`

Here, you will be shaking the harmonic oscillator. That is, the potential will be modified to

$$V_{\text{shake}}(x) = \frac{1}{2}m\omega^2 \big[x - f(t)\big]^2 \tag{3}$$

where

$$f(t) = a\sin\left(k\omega t\right). \tag{4}$$

The questions below will ask you to observe the behavior of the system.

1. First, try varying the amplitude of the modulation a. How does the amplitude of the modulation affect the results?

2. Try varying the frequency of the modulation. As shown in Eq. (4), this frequency is represented by a scaling constant $k\omega$ where ω is the harmonic frequency and k is a proportionality constant you can change in Composer. Qualitatively describe the behavior of the system.

3. The *Scalar Time Trace Plot* nodes on the right of the *For Loop* scope plot the population in each harmonic oscillator eigenstate as a function of time. What states are accessible by this type of modulation?

4. How are the resonance frequencies different?

5. (optional) If you have had any formal training in perturbation theory and Fermi's Golden Rule, you can (and should!) use this to check your results. For example, in each case, to leading order in x, what does the matrix element connecting two states look like? Where does perturbation theory break down? Such physical insight is often invaluable when analyzing control problems like we will do in the exercises that follow.

2 Problem 2: Squeezing the Harmonic Oscillator

Open the file `modulation_spectroscopy_squeeze_2019.flow`

This problem looks the same as Problem 1, but we will be squeezing the harmonic oscillator. In this case, the potential will be modified to

$$V_{\text{shake}}(x) = \frac{1}{2}m\omega^2\big[1 + f(t)\big]^2 x^2 \qquad (5)$$

where, as before, the squeezing function $f(t)$ is given by Eq. (4).

The questions below will ask you to observe the behavior of the system.

1. First, try varying the amplitude of the modulation a. How does the amplitude of the modulation affect the results?

2. Try varying the frequency of the modulation. As shown in Eq. (4), this frequency is represented by a scaling constant $k\omega$ where ω is the harmonic frequency and k is a proportionality constant you can change in Composer. Qualitatively describe the behavior of the system.

3. The *Scalar Time Trace Plot* nodes on the right of the *For Loop* scope plot the population in each harmonic oscillator eigenstate as a function of time. What states are accessible by this type of modulation?

4. How are the resonance frequencies different?

5. (optional) If you have had any formal training in perturbation theory and Fermi's Golden Rule, you can (and should!) use this to check your results. For example, in each case, to leading order in x, what does the matrix element connecting two states look like? Where does perturbation theory break down? Such physical insight is often invaluable when analyzing control problems like we will do in the exercises that follow.

3 Problem 3

Briefly comment on the differences between what you observed in Problems 1 and 2. If you could answer the optional questions, use these answers in your explanation here.