The Quantum Control Challenge

Background

Quantum Control and the Quantum Speed Limit

Outside of a few cases, time-dependent quantum mechanics is very difficult to solve analytically. Additional complexities arise when we try to control a system. For example, if we prepare an quantum system in a state $|\psi_0(t=0)\rangle$ (often an experimentally challenging process in itself), we may want to find an protocol that drives the state to a desired $|\psi_{\mathrm{D}}\rangle$ in some time T with some minimum fidelity F. We define the fidelity just as in Composer, that is

$$F = |\langle\psi_{\mathrm{D}}|\psi(t=T)\rangle|^2. \tag{1}$$

This type of state-to-state transfer is the basis of *quantum control*. [1]

There is another extremely interesting concept in Quantum Control, that is, the *quantum speed limit*, or QSL. The QSL provides a measure of the minimum transfer time to achieve optimal state transfer with unit fidelity. The QSL can be nicely related to the time-energy uncertainty principle $\Delta t \Delta E \geq \hbar/2$ such that (for the simplest cases)

$$T_{\mathrm{QSL}} \propto \frac{1}{\Delta E} \tag{2}$$

and while a detailed proof of this *Bhattacharyya bound* is left to Ref. [1], some interesting insight can be gained.

As we explored in the double-well system, if we initialized a state localized in the left well and moved it close enough to the right well (but not so close that the state was no longer localized to a single well), we could just let the system run, and the state would move periodically between the two wells with some frequency ω. Therefore, if we wanted the state to move from the left to the right well, we could simply bring two wells together, wait a time $T_\pi = \pi/\omega$, then separate them again.[2]

The question is, if we exert some control on the system (e.g. by modulating the barrier height or distance between the wells), can we drive this transfer faster than this? (Answer: probably. Extra imaginary research points if you set this up in Composer and try it!) However, any faster transfer is going to require that we involve states other than the two creating the superpositions between the left and the right well. For the untilted well, the two states we were using were the lowest-energy states of the system, so by definition, if we are going to work with any other states, they will have to be higher in energy. Indeed, the uncertainty in energy ΔE can be related to how quickly we change the state $|\psi(t)\rangle$ to some state orthogonal to it (that is, another state of the system).

Therefore, if we can quickly pump energy into the system to drive it to a higher state, then just as quickly remove that energy from the system to drive it back down to our desired state, we are in business. This is shown in Ref. [1] for the two-level system, where the optimal control is shown to be a series of two infinitesmally short, infinitely high control pulses separated by some free evolution time where the control is turned off.

Quantum Control of Bose-Einstein Condensates

In this challenge exercise, you will explore a very interesting problem in quantum control that has been applied in modern ultracold atom research [3, 5, 4]. There are a number of open questions related to this problem that we do not have a good feel for, so in many ways, this exercise is very open-ended. We will provide you with a sketch for how you should go through the flowscenes and some guiding questions, but the results and their interpretation will largely be up to you.

[1] With some exceptions, for example, when we want to find a target unitary operation $U(t_0, T)$. However, here we will consider only state transfer.

[2] If you are familiar with Rabi flopping, this would correspond to a π-pulse.

For this exercise, you will explore four different scenarios, each with the same goal: starting from the ground state of a given potential, modulate the potential and drive the state into the first excited state. First, you will explore harmonic and anharmonic control. Just as in the Shake and Squeeze exercise, your control will be the modulation (shaking) of the potential.[3]

Up until now, you have simulated a single particle (like a single atom in a well). Thus, the second element of the exercise will add a nonlinearity to the potential due to atom-atom interactions in a Bose-Einstein condensate, the coldest state of matter we know of.[4] This is best illustrated by looking at the form of the two Hamiltonians. For a given potential $V(x,t)$, the single-particle Hamiltonian $H(x,t)$ takes the familiar form

$$H(x,t) = \frac{p^2}{2m} + V(x,t). \tag{3}$$

However, if we add the nonlinearities due to the interactions of ultracold atoms with one another and make a few approximations (that are reasonably good in most situations), we obtain the so-called Gross-Pitaevskii equation (GPE) [2]

$$H(x,t) = \frac{p^2}{2m} + V(x,t) + \beta|\psi(x,t)|^2, \tag{4}$$

where the parameter β controls the atom-atom interaction. This parameter is dependent on the atom density, the atom-atom scattering length, and the atomic mass. This nonlinearity is interesting in that it modifies the effective potential seen by the atoms, and as a result, controls that work for a single particle may fail when one considers a BEC.

The flowfiles

Here, you are provided with four sets of three flowfiles. The four sets are as follows:

- single particle, harmonic potential
- single particle, anharmonic potential
- BEC, harmonic potential
- BEC, anharmonic potential

For each of these, you will be able to control the potential modulation via

- A graph with points that you can click-and-drag
- A function that you write directly into Composer
- Optimal control with the GRAPE algorithm

In this challenge, please leave the potential parameters as-is. That is, don't change the values that go into the *Potential* nodes. However, when exploring the BEC levels, you may change the values of the nonlinearity parameter β. In addition, the spatial nodes and the temporal discretization dt has been set and should remain fixed. However, you can (and should!) change the final time dt. For simplicity, I list the parameters that may be changed:

- The final control time T
- The BEC nonlinearity parameter β (where applicable!)
- The control graph or control function

You should have seen everything before, with the possible exception of the flowfiles with the control function. For these flowscenes, you can directly enter a modulation function into the box labelled *Scalar Expression* in the time evolution *For Loop*. Note that you have been given variable *E01* and *E12*, the energy differences between the ground state and first excited state as well as the first two excited states,

[3] Think: why aren't we squeezing the potential if we want to excite the atom into the first excited state?
[4] We make these down in the lab!

respectively. These variables (and the final time *tf*) can be entered into the modulation function box. Everything else should be hard-coded—that is, enter the number directly!

Syntax in the modulation function box is like that used in most programming languages. If you cannot get your favorite common function to work, ask and we can help with syntax. See Figure 1 for an example of syntax that Composer can recognize.

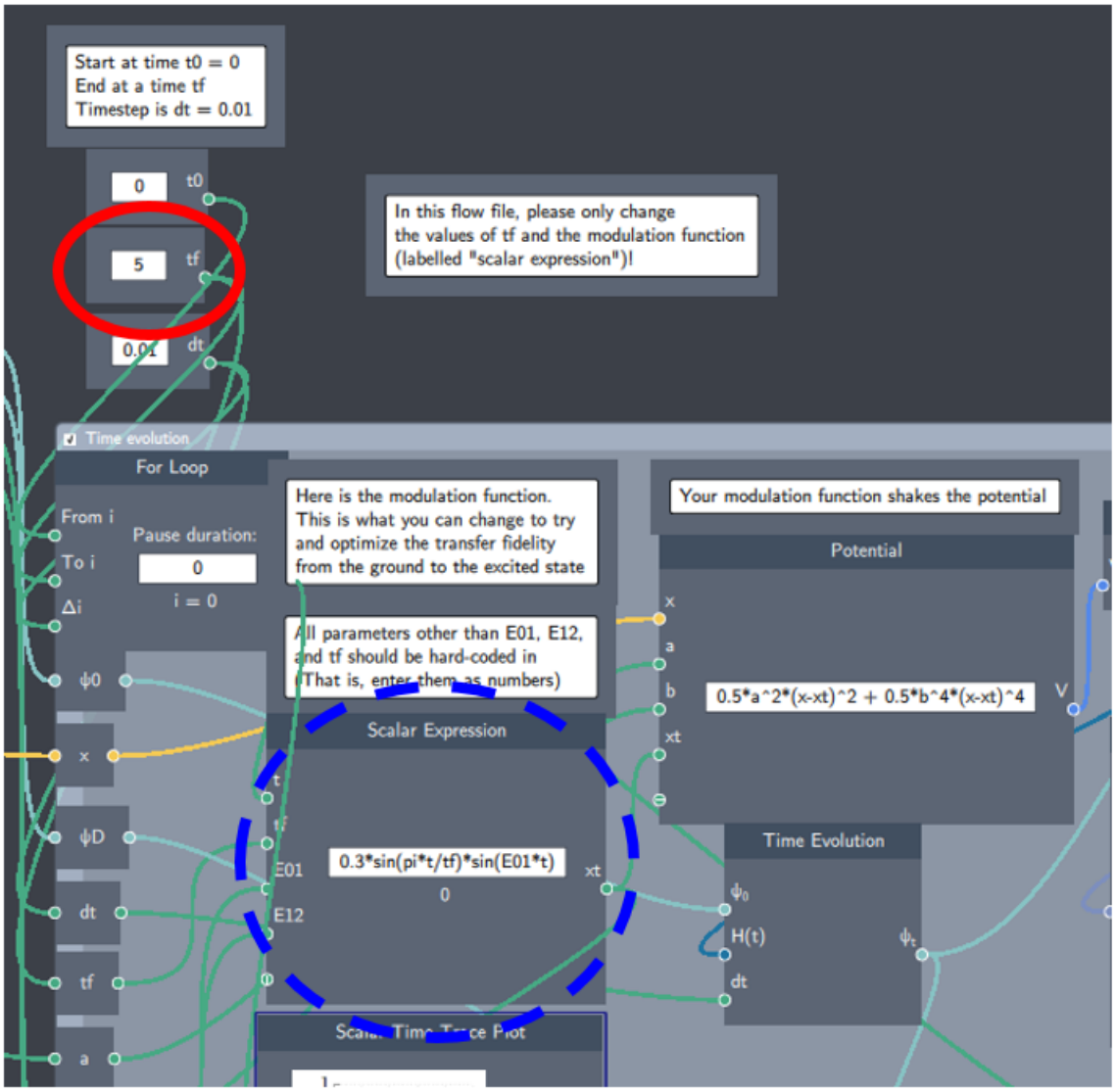

Figure 1: A snapshot of part of a Composer flow file that you will be using for parts of this challenge. This Composer flowscene allows you to control the modulation function by directly entering a command into a *Scalar Expression* box (blue, dashed circle) in the time evolution scope. You can also change the final time *tf* (red circle).

Below, we give you some guiding questions to explore as you work through the challenge.

The challenge

For this challenge, you should explore the following things, in the following order. Note that you cannot answer all of these questions in your report, so you should pick an interesting thread or two and explore them in detail. We much prefer an in-depth analysis of a few compelling bits of the problem than a shallow analysis of many aspects of the problem.

1. Start with the single-particle flowscenes. Play with both the harmonic and anharmonic potentials using the draggable graph and the function input scenes. Don't use the GRAPE optimization flowscenes yet. Consider the following questions:
   - What is the best fidelity you can achieve in each case (for $tf = 5$).
   - What is the minimum time in which you can obtain $F > 0.5$? $F > 0.75$? $F > 0.9$?
   - Which control method is easier for you to work with? Why?
   - What strategies work for each case? Which do not?
   - Why is harmonic control so much harder than anharmonic control?
2. Now play with the BEC problems. Consider:
   - Do the same controls work for the BEC as for the single-particle?
   - How does your answer to the above change as you change β?
   - Is harmonic control easier or harder with the BEC?
   - What happens if $\beta < 0$? What does this correspond to physically?
3. Once you have given the non-GRAPE flowscenes a try, you can move on to the GRAPE-based flowscenes.
   - Does harmonic control work for the single-particle case?
   - What about the BEC case? If so, for what values of β does it work?
   - How are controls different for the single-particle and BEC case, given a similar final time?
   - Map out the QSL for the BEC and single-particle cases. Note any differences.
   - Does the QSL change dramatically if you change β?

Reporting your results

As stated in the overview, please write up your most interesting results in a cohesive 2-4 page report. Describe your work and include pictures! A LaTeXtemplate will be provided for you.

Remember: we don't know the exact answers to many of these questions, so part of the game is to teach us something! In many ways, this is truly original research.

Finally, please think about the following: Are there any other problems you would like to explore in Composer? We are always open to new and exciting ideas!

References

[1] H. L. Mortensen. Fantastic control functions and how to find them, 2017.

[2] C. J. Pethick and H. Smith. *Bose-Einstein Condensation in Dilute Gases*. Cambridge University Press, 2008.

[3] W. Rohringer, R. Bücker, S. Manz, T. Betz, Ch. Koller, M. Göbel, A. Perrin, J. Schmiedmayer, and T. Schumm. Stochastic optimization of a cold atom experiment using a genetic algorithm. *Appl. Phys. Lett.*, 93:264101, 2008.

[4] S. van Frank, M. Bonneau, J. Schmiedmayer, S. Hild, C. Gross, M. Cheneau, I. Bloch, T. Pichler, A. Negretti, T. Calarco, and S. Montangero. Optimal control of complex atomic quantum systems. *Nat. Sci. Rep.*, 6(34187):34187, 2016.

[5] S. van Frank, A. Negretti, T. Berrada, R. Bücker, S. Montangero, J.-F. Schaff, T. Schumm, T. Calarco, and J. Schmiedmayer. Interferometry with non-classical motional states of a Bose-Einstein condensate. *Nat. Comm.*, 5:4009, 2014.