Building a quantum system in Quantum Composer

Part II: Spectra

2 Potentials and spectra

2.1 Thinking about simple polynomials

Let's look at the properties of a simple function, $f_l(x) = |x^l|$, where l is a parameter. We take the absolute value of the function to give it a bowl-like shape, even for odd l. If we didn't, the form of the potential would be open and the particle wouldn't be confined. (*Confined* means the particle is mostly held within the potential well.)

Now restrict the domain of f_l to $x > 0$ (this makes differentiating the function easier). For $l = 1, 2, 3, 4$ differentiate f_l [*Hint: if* $x > 0$, $|x^l| = x^l$]. Think about the how the gradient of the function in the regions $0 < x < 1$ and $x > 1$ changes as l is increased. Sketch the functions on one set of axes.

Finally, consider what f_l looks like for very large l. Sketch this function.

2.2 Investigating potentials

Before jumping into the simulation, we should define some notation. Let n be the number of the energy level, starting with zero for the lowest energy, one for the second lowest, and so on. Then E_n is the energy of level n.

Open the file `Exercise1.flow` in *Quantum Composer*. Locate the box labelled **Potential**. It looks like Figure 1.

You have already seen what the energy levels for the harmonic potential, `0.5*a^2*x^2`, look like. Here, using the notation just introduced, $E_n \propto n$. Investigate the potential of the form f_l for very large l. To do this, type `infinite(-a,a)` in the **Potential** box. This creates a potential which is zero between $-a$ and a, and infinite everywhere else. Inspect the spacing of energy levels, you should notice that as n is increased, the energy difference $E_{n+1} - E_n$ increases. In fact, the position of the n^{th} energy level is such that $E_n \propto n^2$.

Plot the potentials $a|x|$ (use the `abs` function) and a^2x^4. Again, look at the spacing of energy levels. Try to create a ranking of the four potentials based on this spacing. Without using *Quantum Composer*, place the potentials $|x^3|$ and $|x^5|$ within your ranking, using what you worked out about these functions in the previous section.

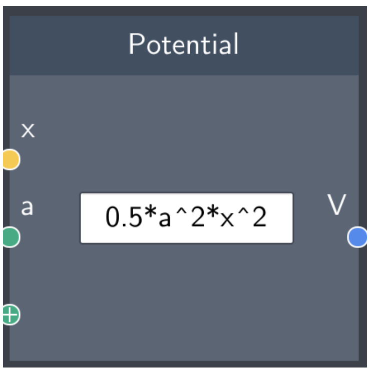

Figure 1: The **Potential** box.

A challenge: l does not have to be restricted to positive integers. Try locating $\frac{1}{x}$ in relation to your ranking. This potential is another example of a potential that is practically useful — it describes the potential energy of an electron in the Bohr model of the atom, proposed by Danish physicist Niels Bohr in 1913.